

# Learning Trajectories in Math for *All* Young Children

Douglas H. Clements and Julie Sarama

**Building  
Blocks**



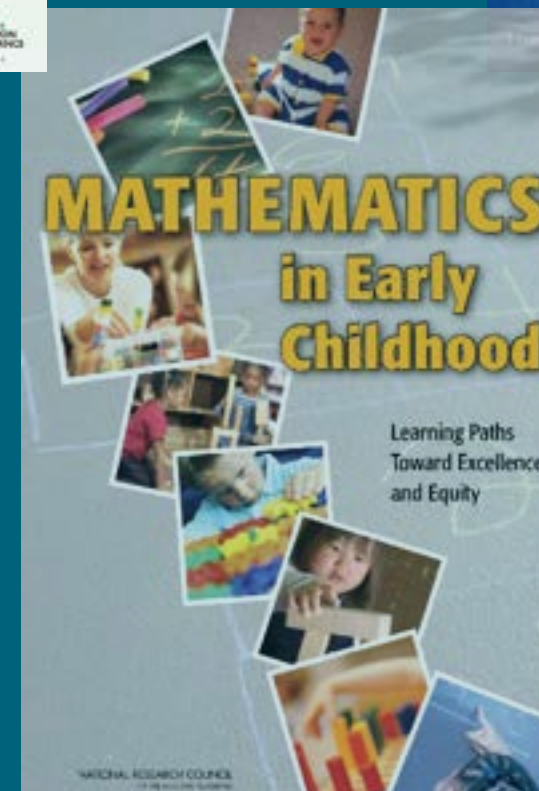
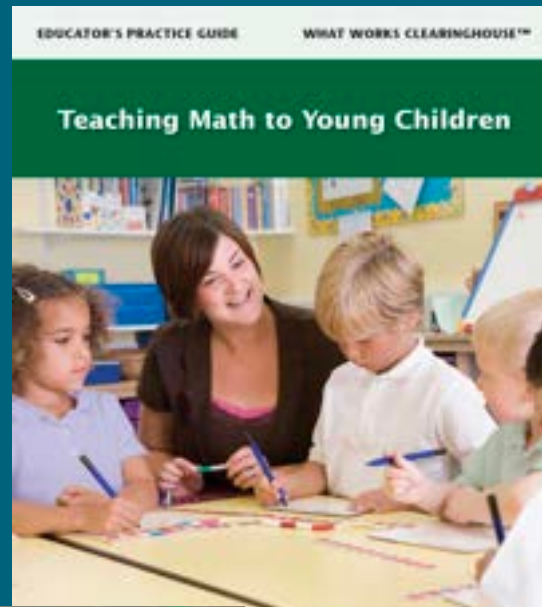
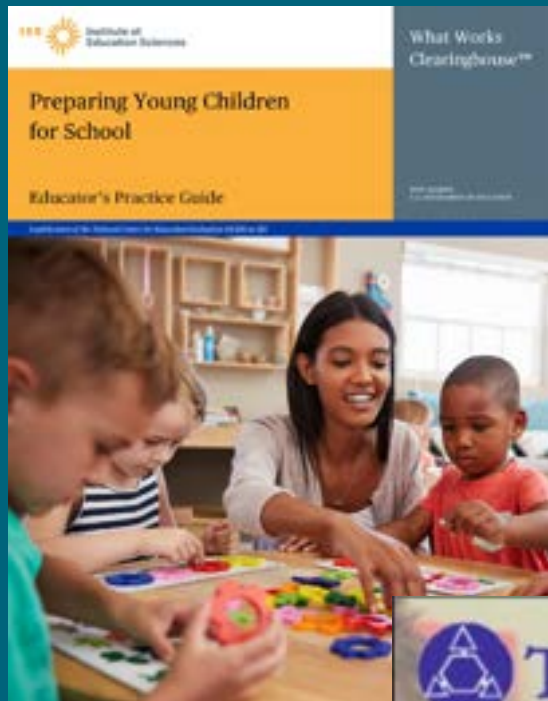
**Marsico Institute**  
FOR EARLY LEARNING & LITERACY



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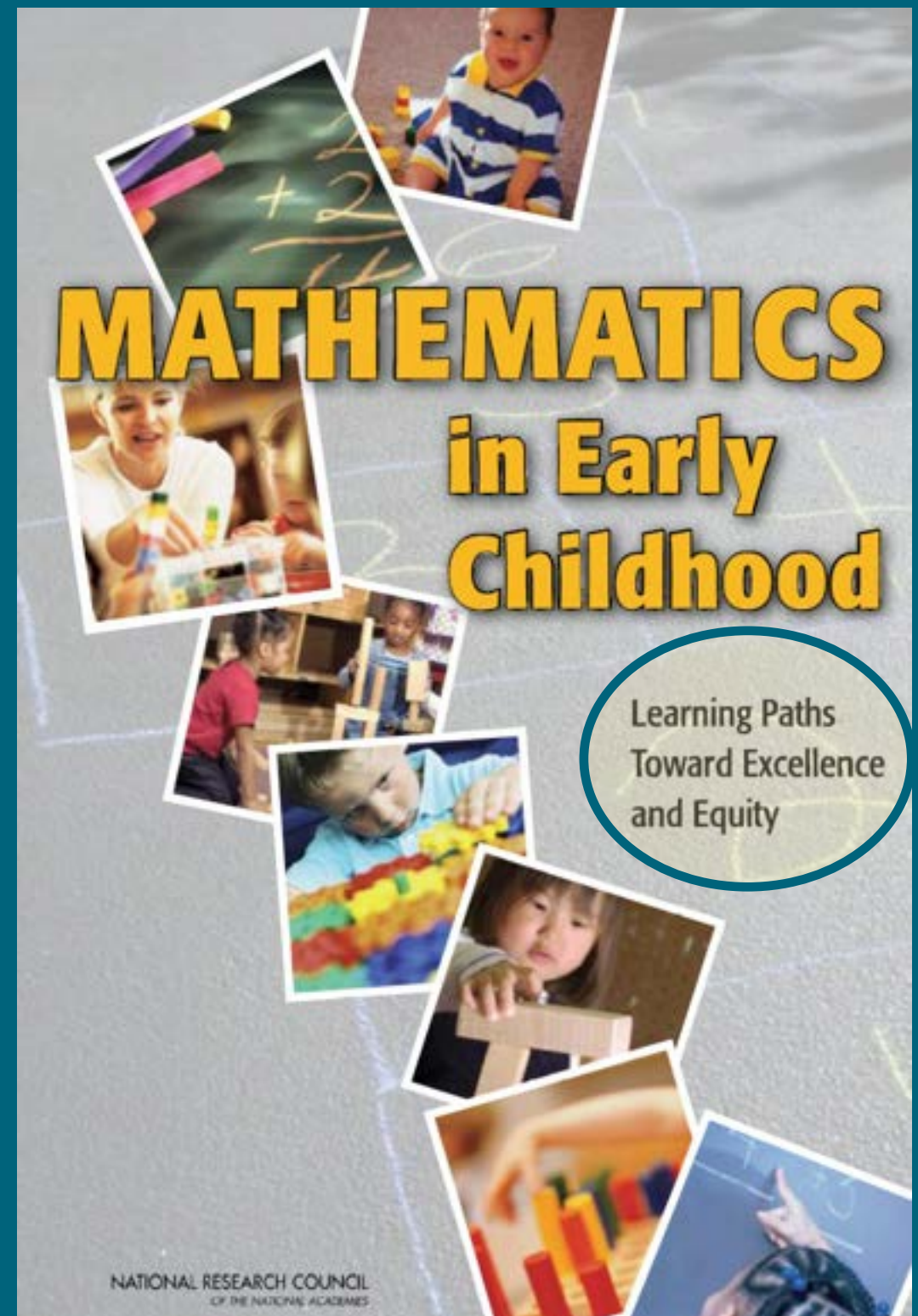
# We Know a *Lot*

About how children *think and* learn about math... and teaching *all* children





# We Know a Lot... about Teaching with Learning Trajectories!



# We Know about...Learning Trajectories!

FROM COGNITION TO SCALE UP



GOAL



DEVELOPMENTAL  
PROGRESSION



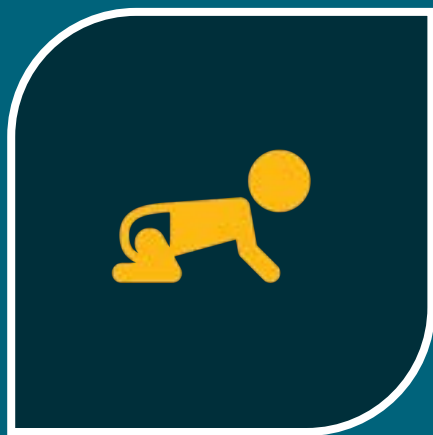
TEACHING  
PRACTICES



# Learning Trajectory as a Garden Path



# Learning Trajectories are *Asset-based*



Development progression what  
*they know and can do*



Teaching practices and activities  
*builds on those, leveling up:  
math their way.*

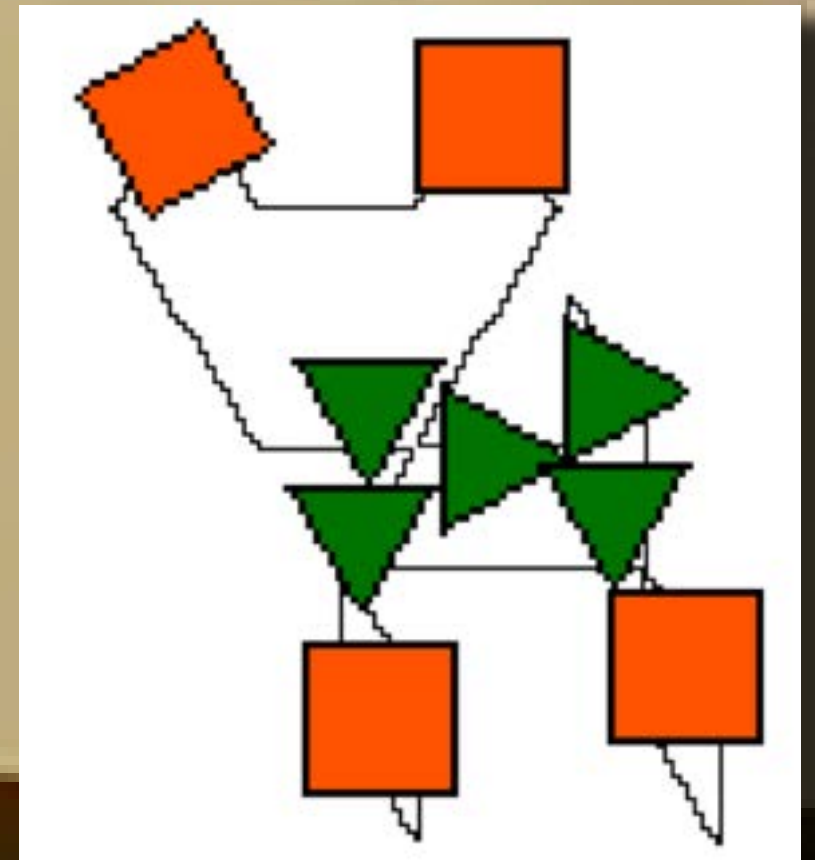


# LT for Composing Geometric Shapes

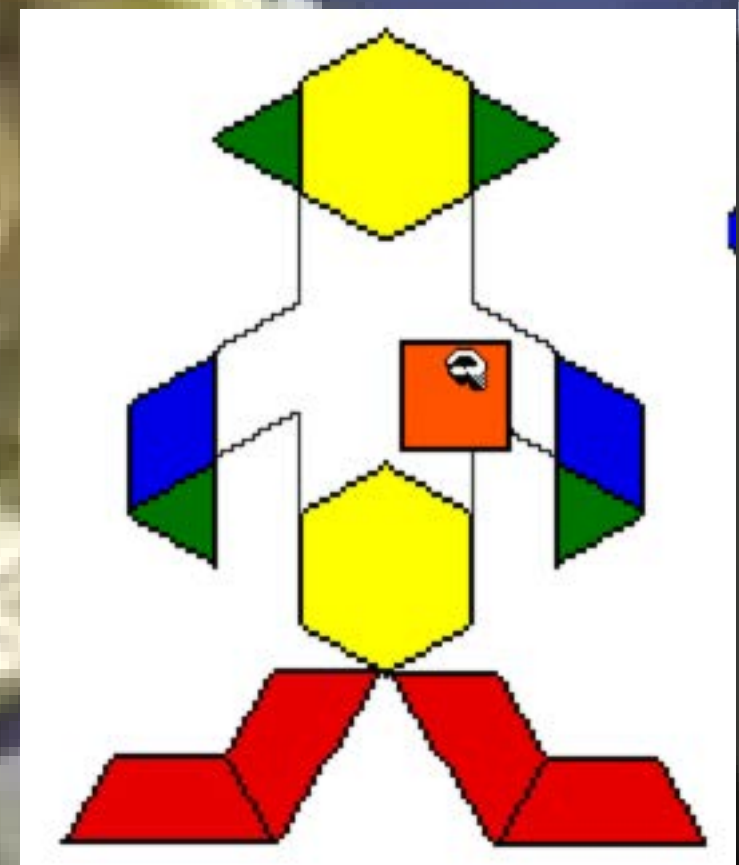




# Separate Shapes Actor: Foundations









Shape Composer

# Instruction



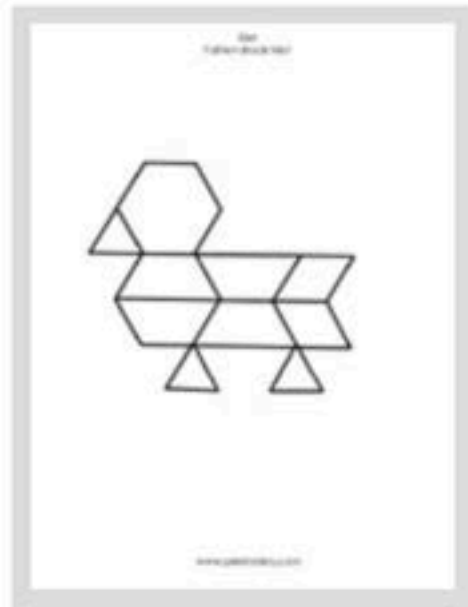




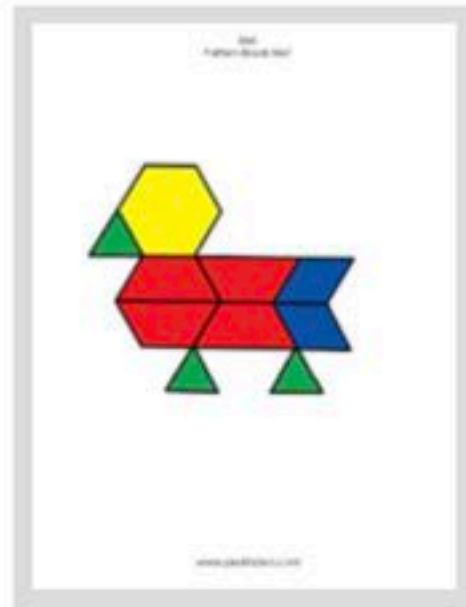
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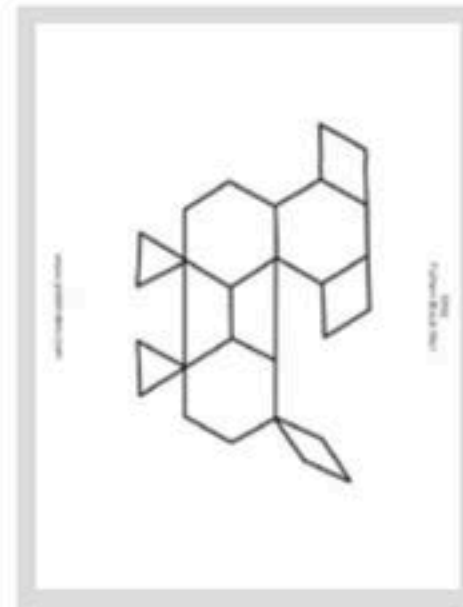
# If levels...put them all out? Hmmm...What levels?



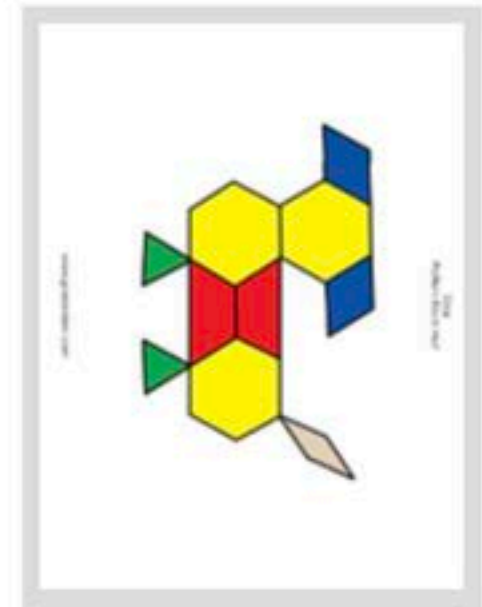
Duck



Duck



Dog



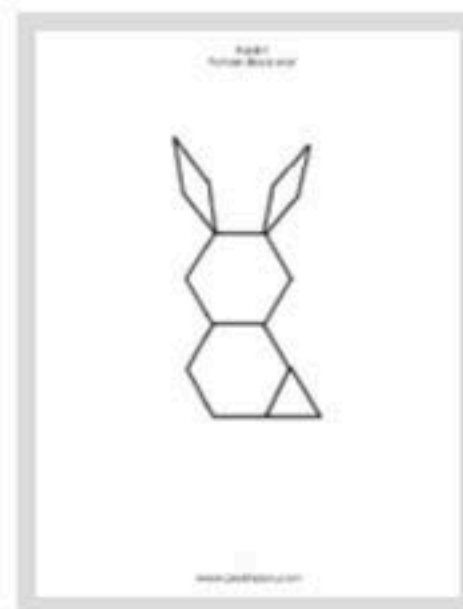
Dog



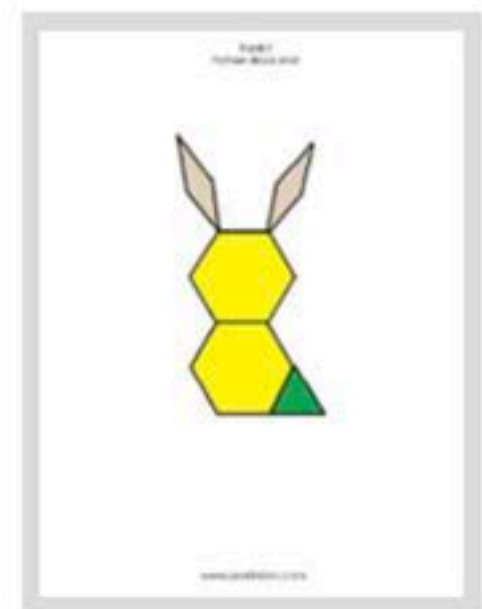
Cat



Cat



Rabbit



Rabbit

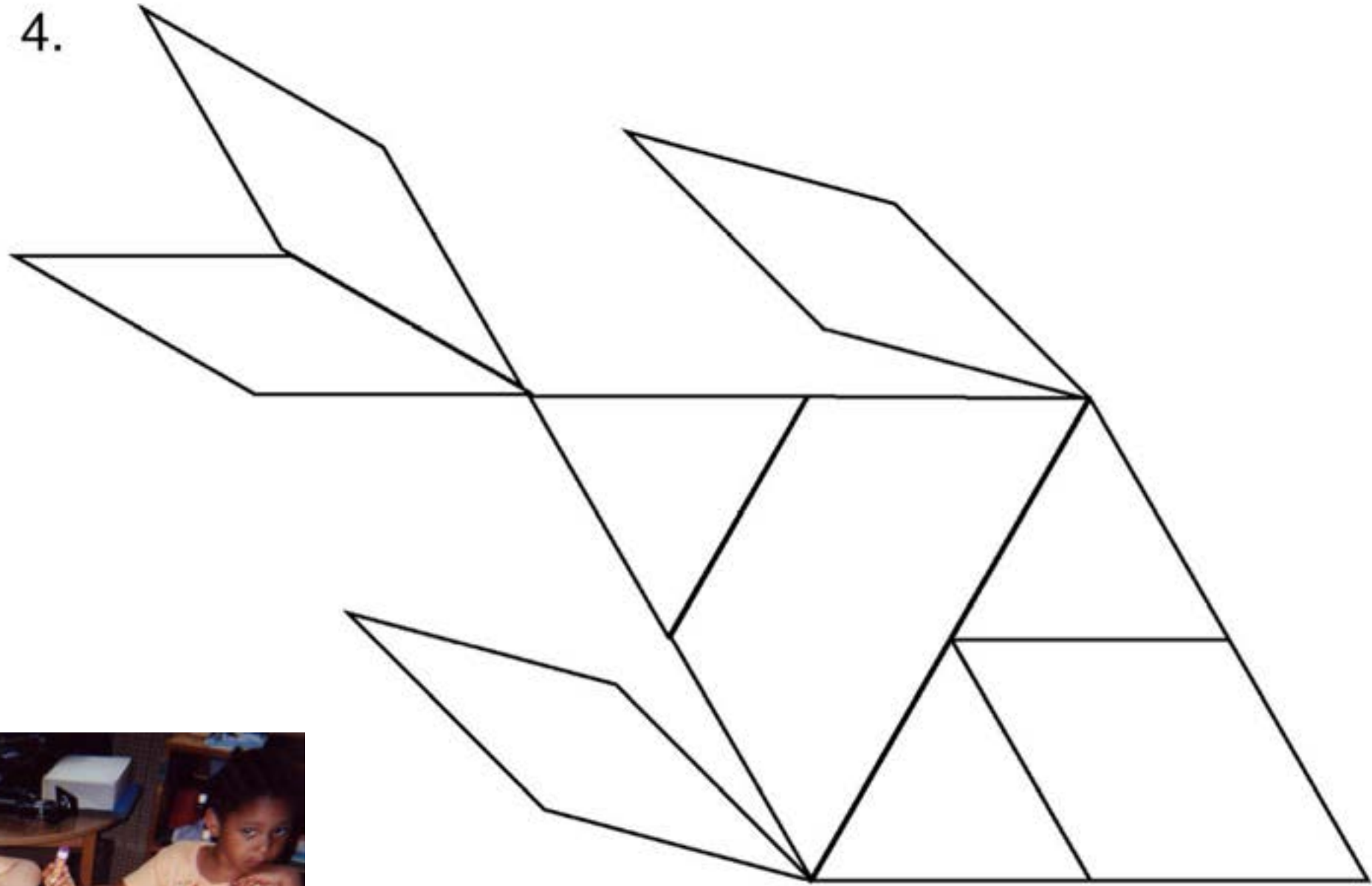




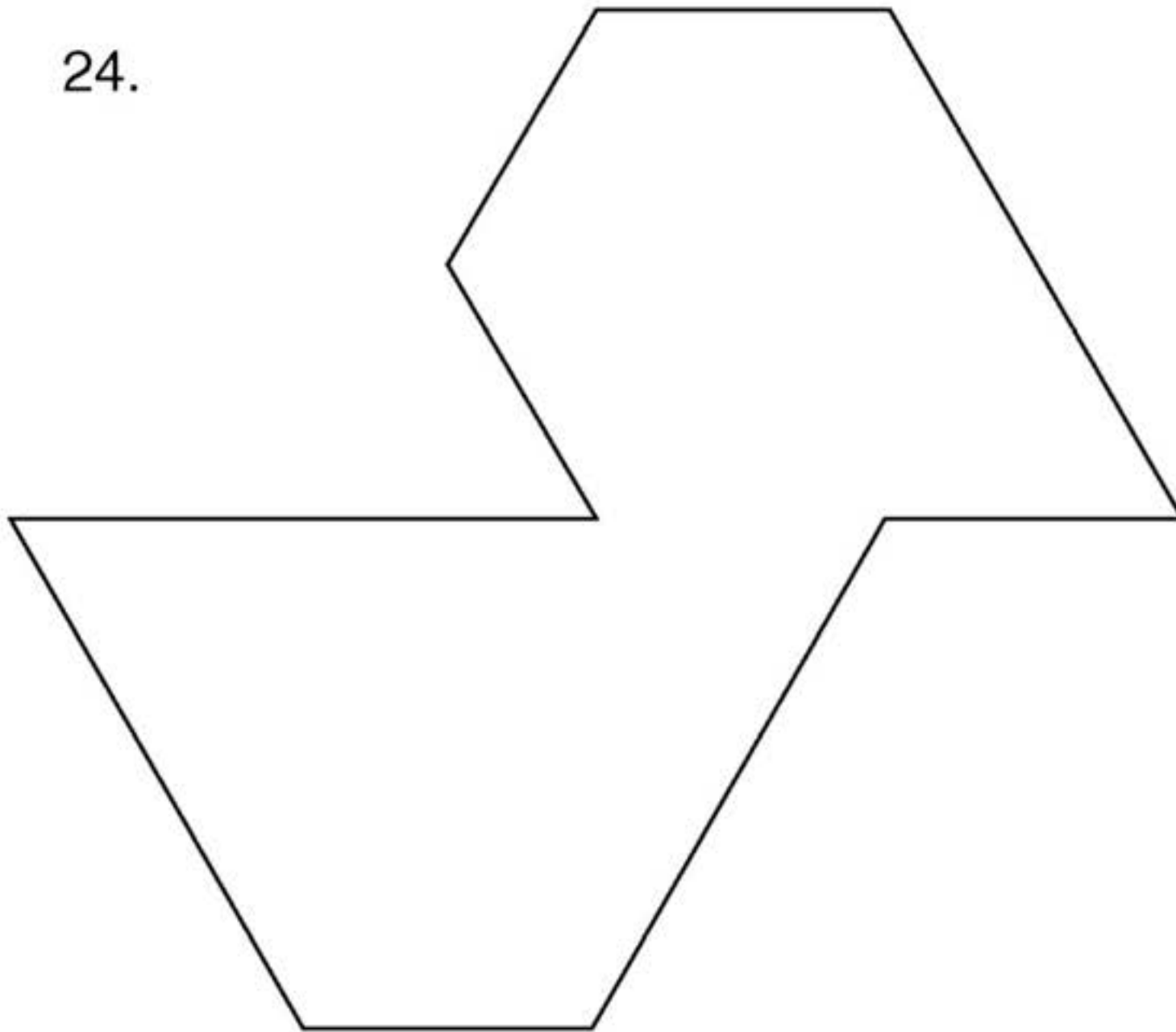




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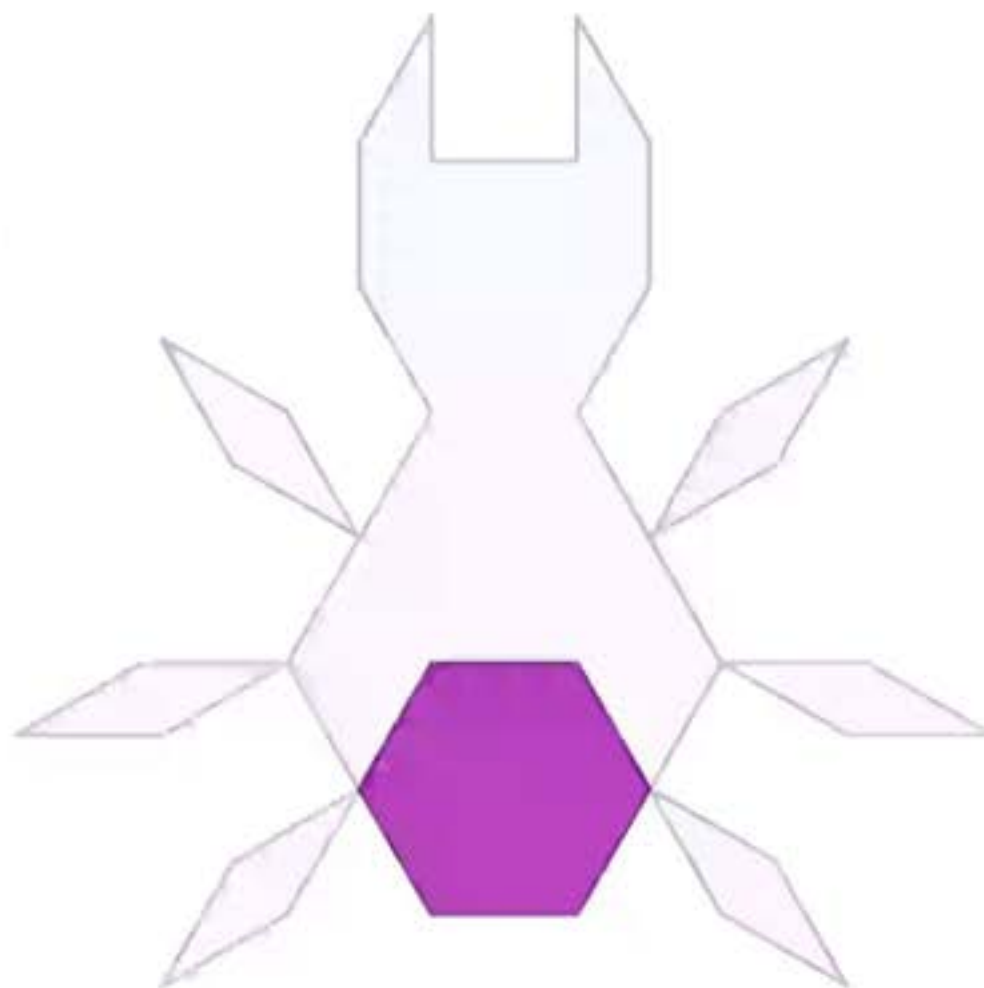
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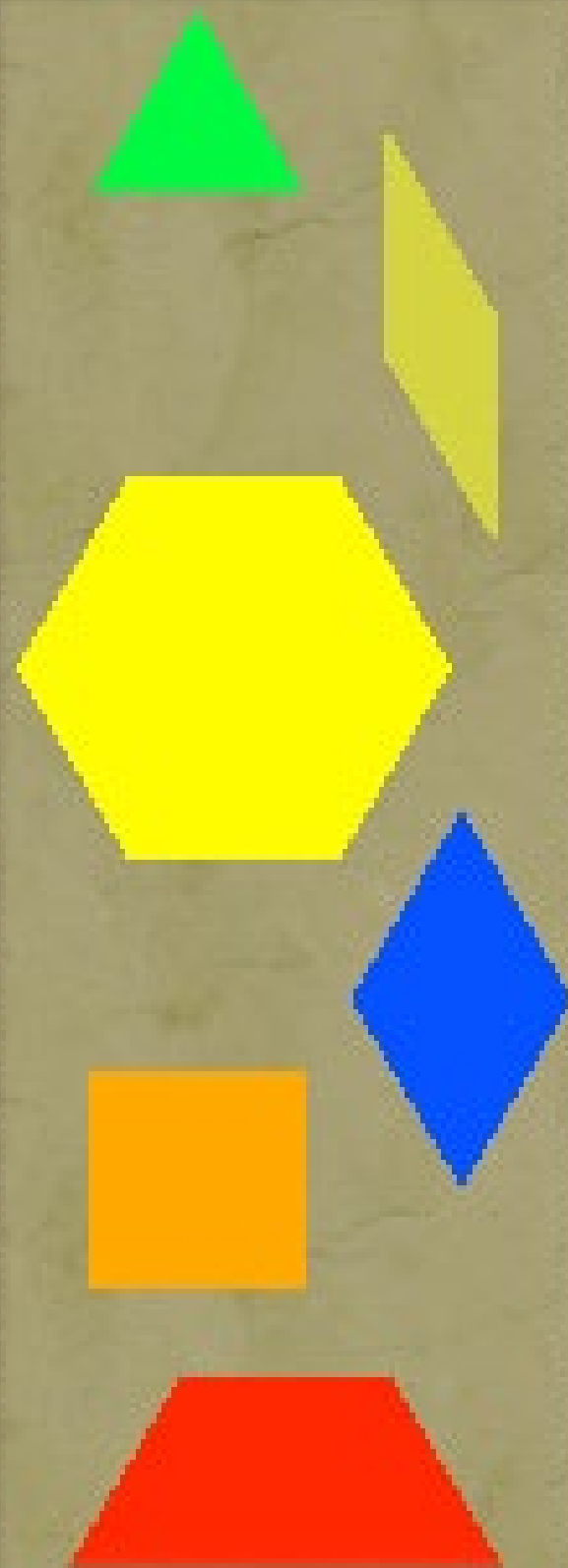
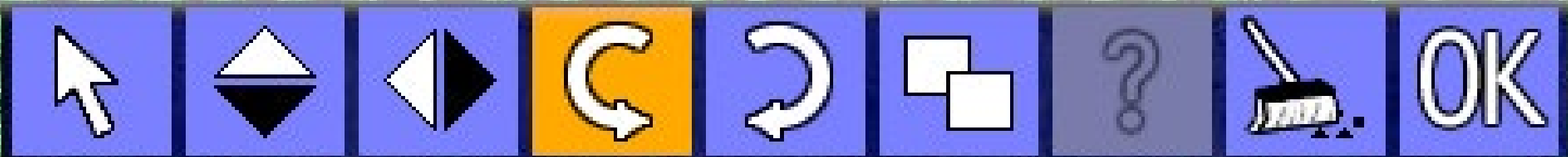
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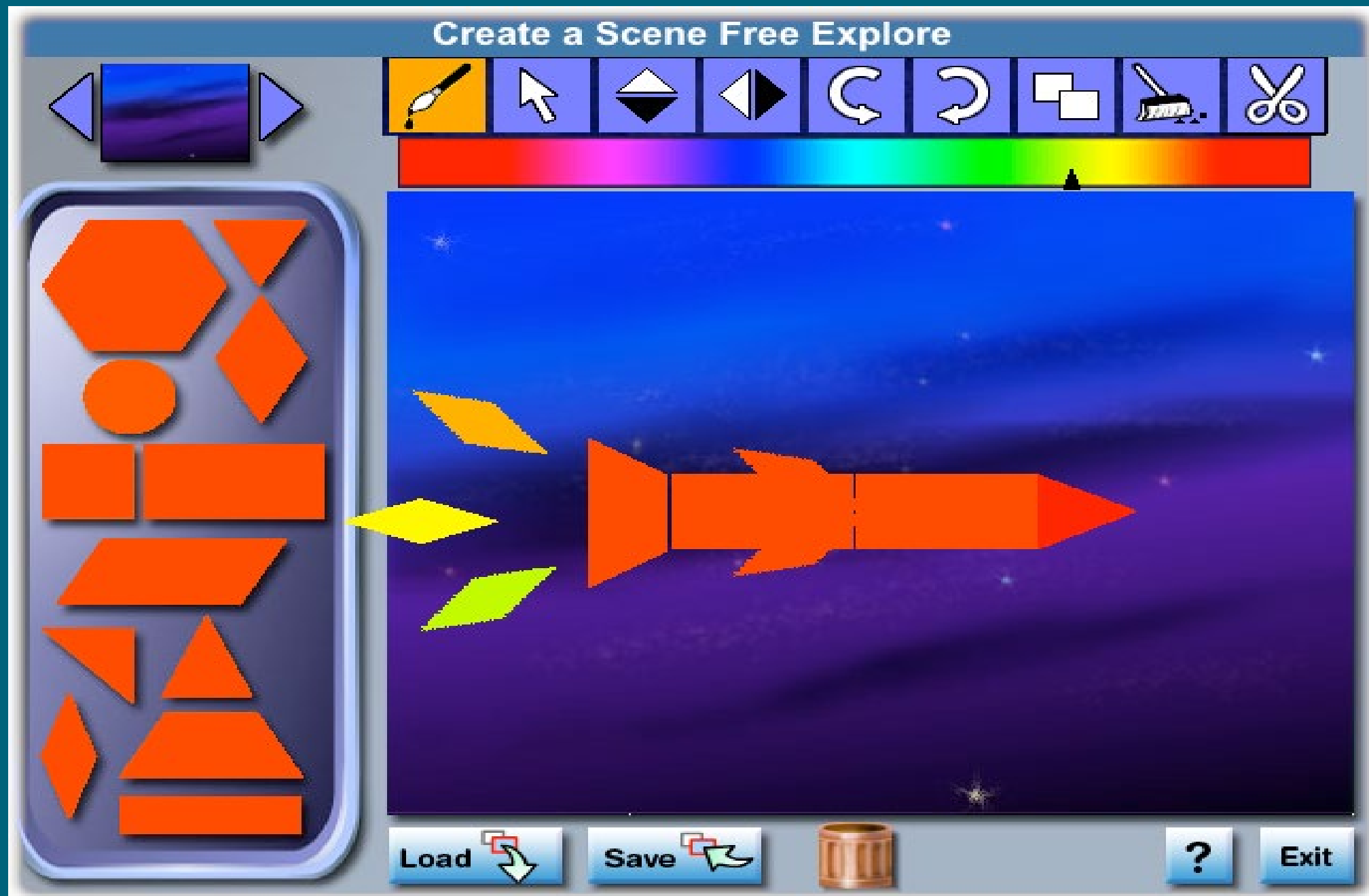
OK



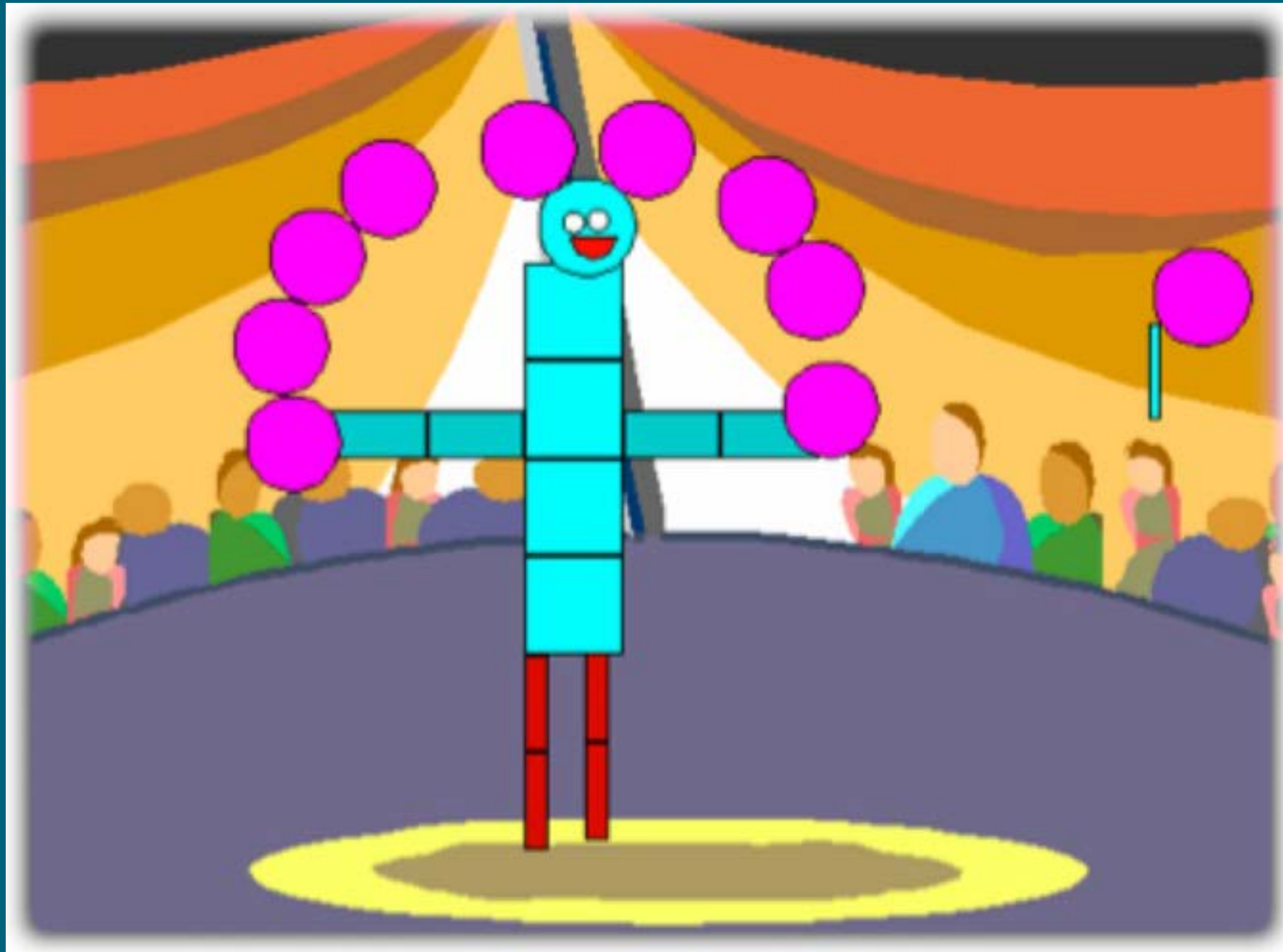




# Create A Scene



# Create A Scene









# **The Implementation of Tangram Puzzle to Enhance Creativity of First Grade Elementary School Students**

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## **1. Abstract**

The development of knowledge and technology is advancing rapidly in this modern era. One of the 21st-century skills that students must possess is creativity. This research aims to: (1) describe the implementation of the tangram media in enhancing creativity in geometry subjects in the first grade of elementary school, and (2) determine the improvement of students' creativity through the application of the tangram media in geometry subjects in the first grade of elementary school. This research adopts a classroom action research with a model developed by Kemmis and McTaggart. The



# 3D Shape Composition with Blocks

- Why an LT at all?
- Open exploratory play *and* solving semi-structured and well-structured problems, with intentional teaching—increased complexity
- Based on *learning trajectories*



# Block Building

- Foundation for achievement in math
- Sophistication of block building in pre-K:
  - predicts standardized math scores in grade 7
  - predicts all high school assessments...
    - number of mathematics courses taken,
    - number of honors courses,
    - advanced math courses taken and grades.



# LTs 3 Parts

1.Goal

2. Developmental Progression

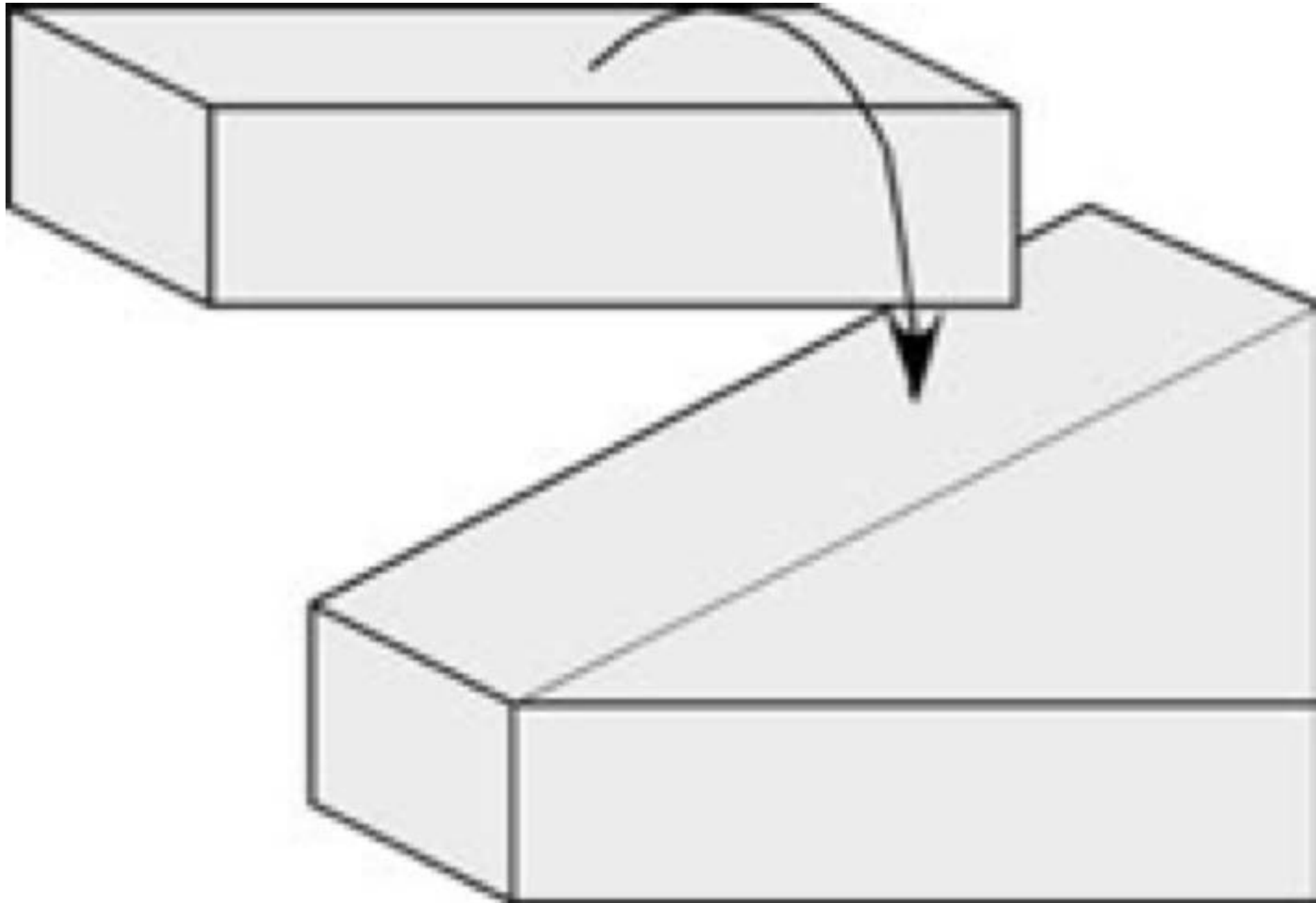
3. Instructional Activities



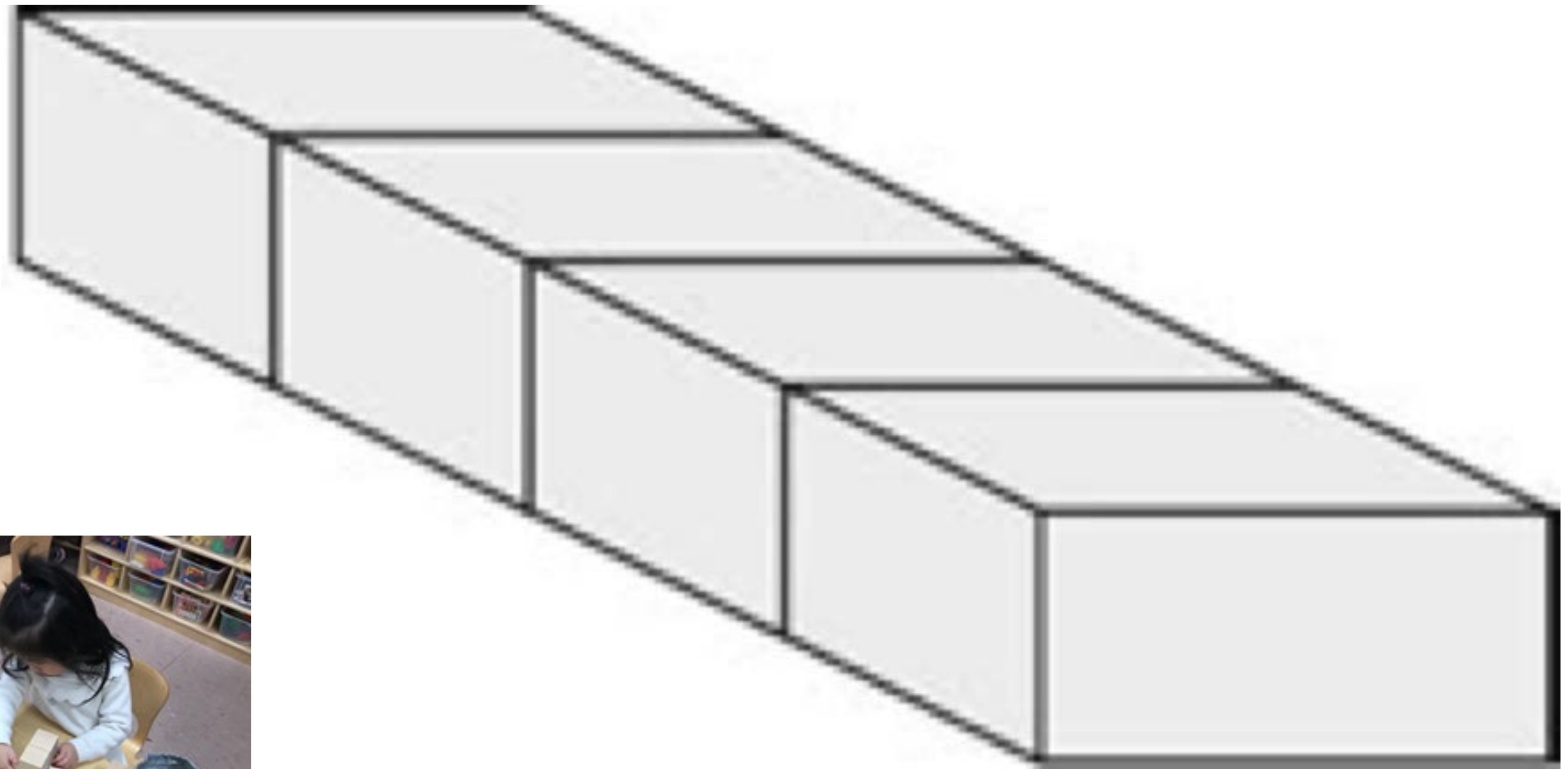
# 3D Composition

- Foundations of Composer: places blocks randomly or individually

# Stacker (1 year, approx.)

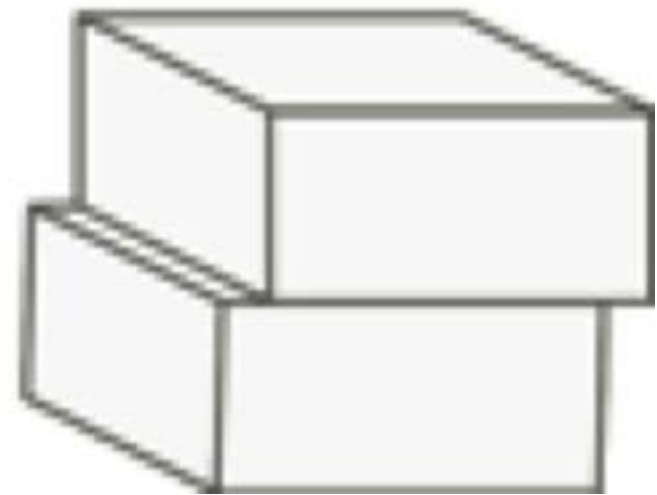
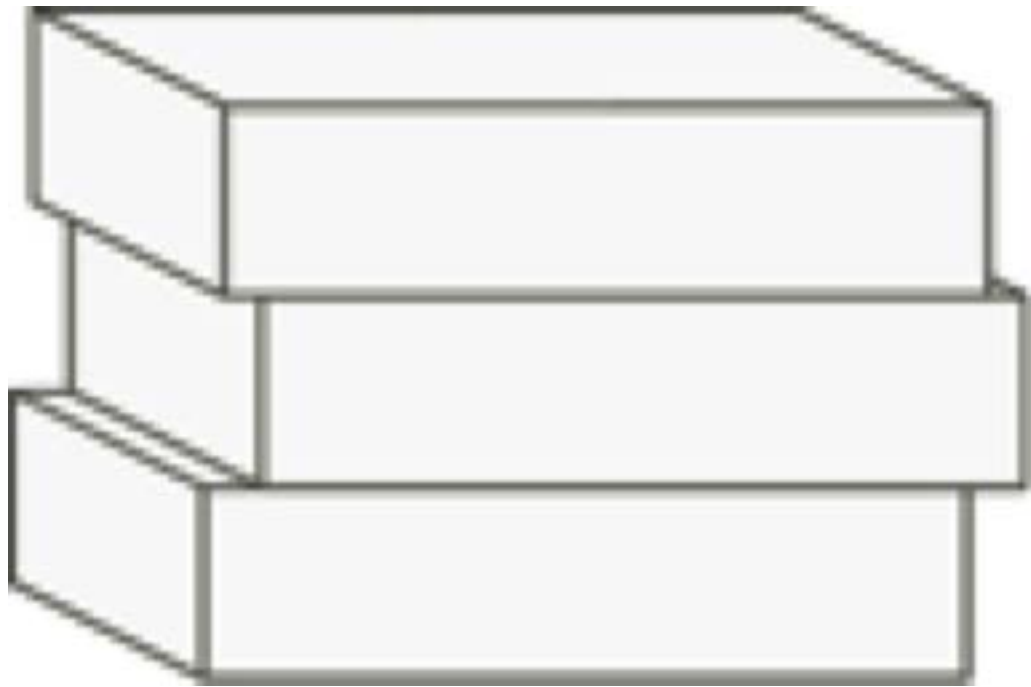


# Line Maker (1.5 years)



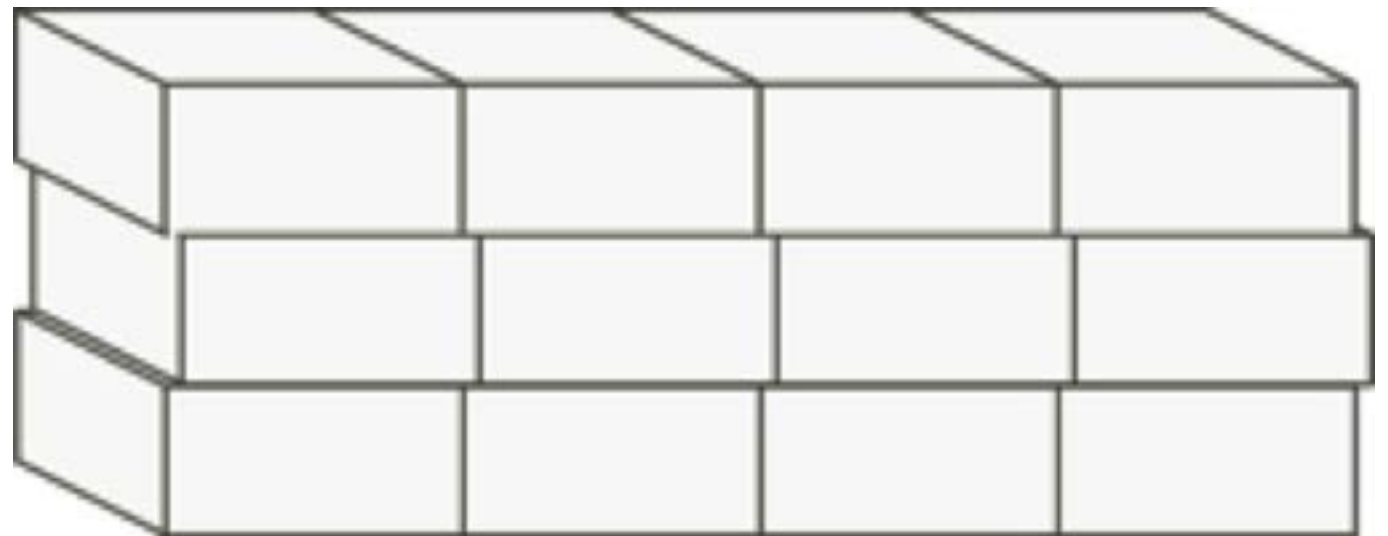
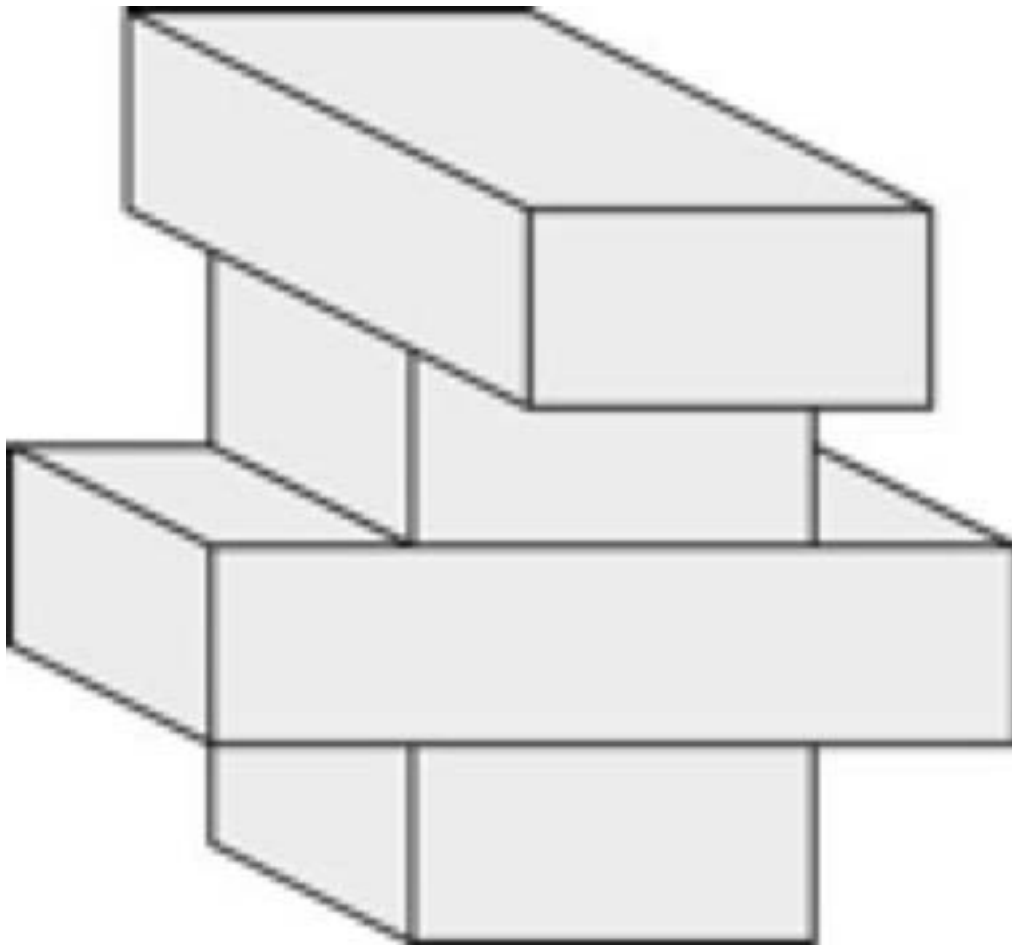


# Same Shape Stacker (2)



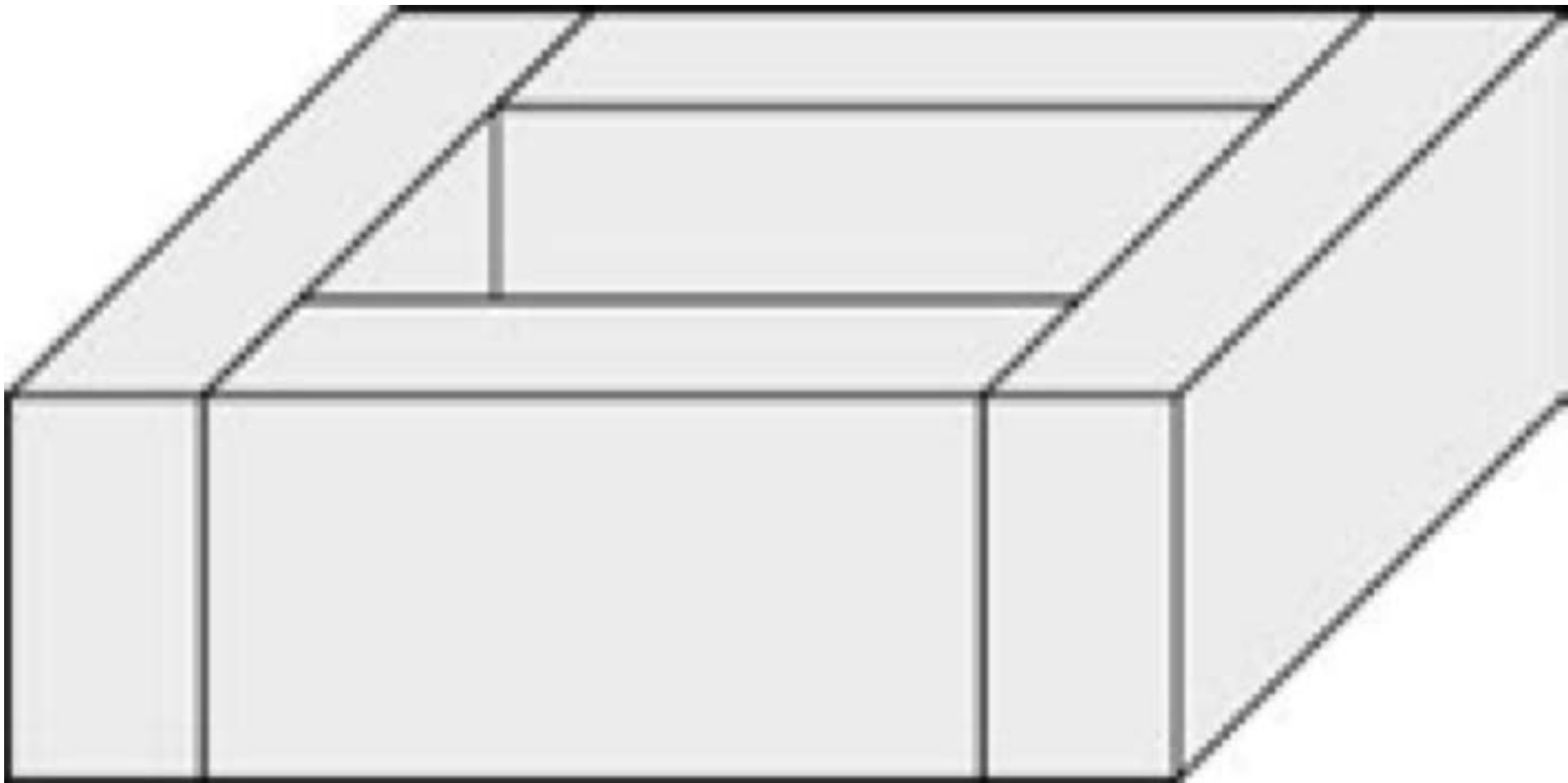


# Piece Assembler (2)

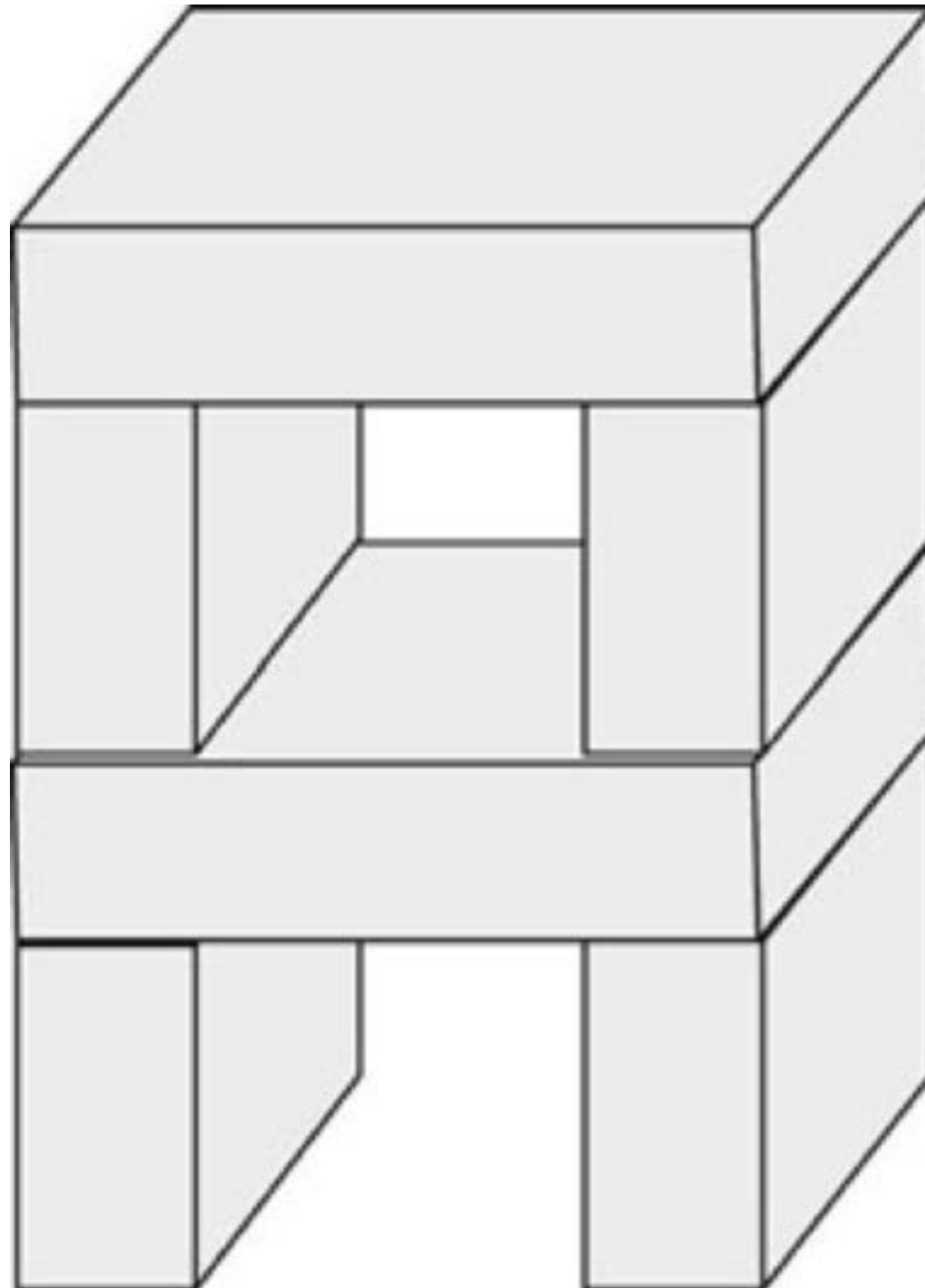




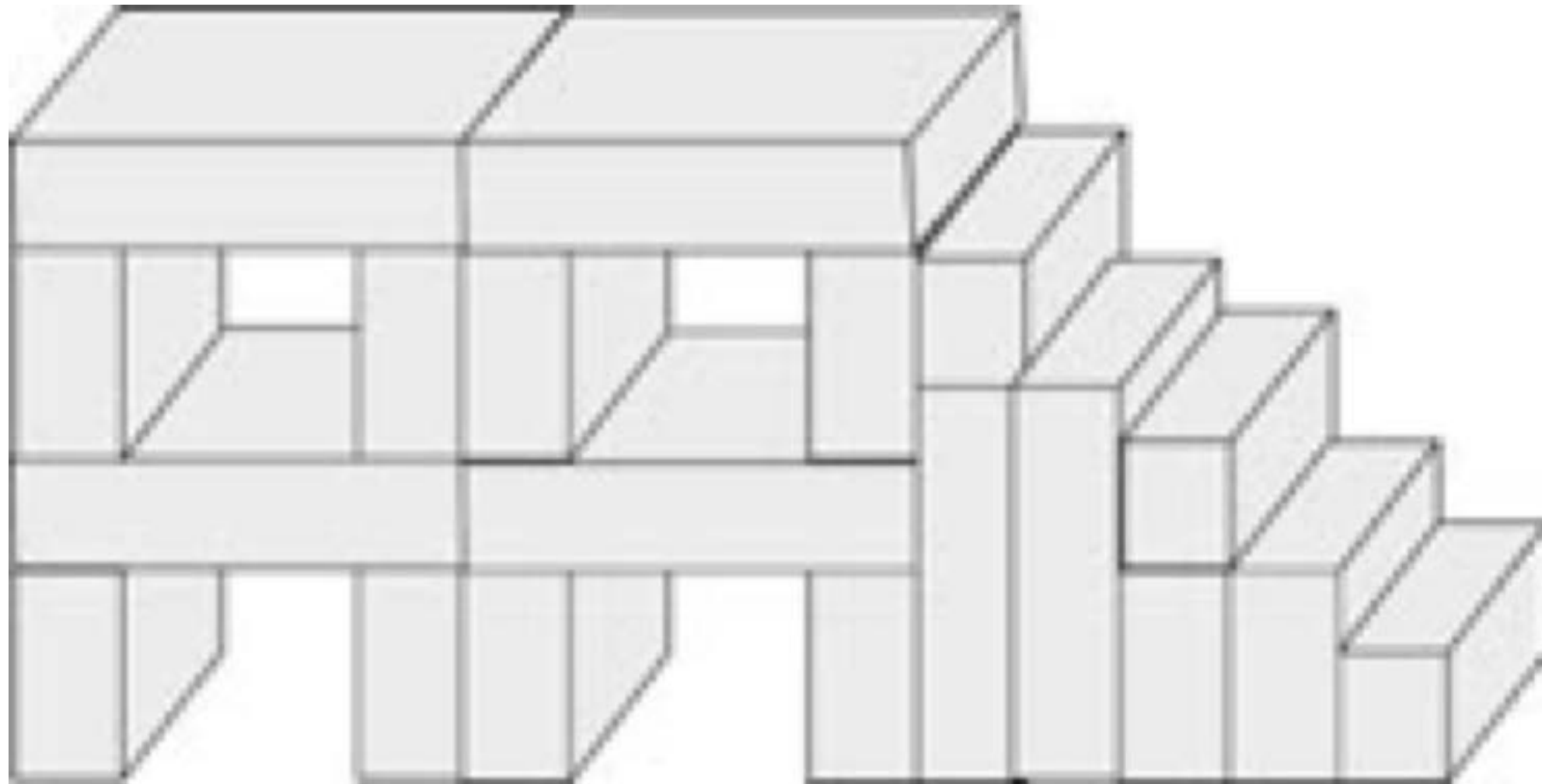
# Picture Maker (3-4 years)



# Shape Composer (4-5)

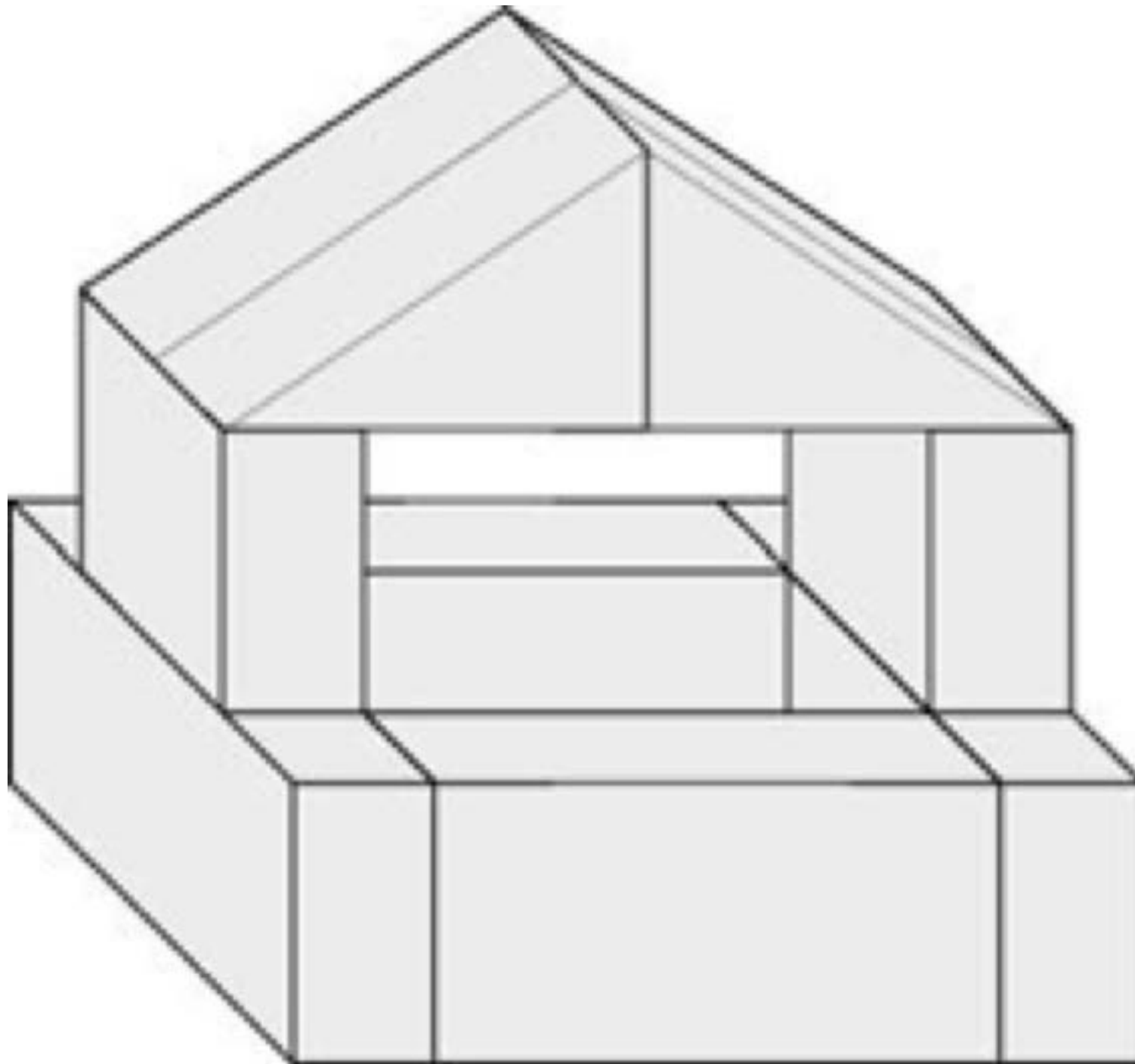


# Substitution Composer and Shape Composite Repeater (5-6)





# Shape Composer—Units of Units (6-8)



# LTs 3 Parts

1.Goal

2. Developmental Progression

3. Instructional Activities







# Facilitating Exploration of Objects

- In general:
  - Facilitating exploration
  - Supports (verbal and nonverbal) to encourage hands-on exploration.
    - Verbal supports — Spanish and English
- In math:
  - Spatial
  - Stacking





# Shape Composer (3D)

Composing 3D Shapes

Shape Composer (3D)

Go to Level

Composes shapes with anticipation, understanding what 3D shape will be produced with a composition of 2 or more other (simple, familiar) 3D shapes. Can produce arches (with vertical interior space), enclosures (with internal horizontal space), corners, and crosses systematically. Builds enclosures and arches several blocks. Later in this level, children add depth to make 3D structures, and they add roofs across structures multiple blocks high (but they may have no internal spaces).

## ACTIVITIES

## You may see this:

1

Children build structures using 3D shapes.



## Other Examples:

- A child makes a "zoo" by combining blocks lining the floor for elephants, walls around tigers, and arches for smaller animals to live in and on.



# Making Composing 3D Shapes Accessible During Routines and Transition

[LT]<sup>2</sup>



- Building learning into existing routines and transitions is an effective way to engage children through repetition and predictability (Jenning, Hanline, & Woods, 2012). By incorporating ***Composing 3D Shapes*** into everyday routines and transitions, the learner is offered multiple opportunities to practice these skills throughout authentic and diverse environments.

## Embedding Composing 3D Shapes into Routines

Centers/Free Play	<ul style="list-style-type: none"> <li>Set up multiple opportunities for children to engage with Composing 3D Shapes <b>during free play or classroom centers.</b></li> <li>Allow children to lead the play, observing, narrating, and modeling when appropriate. This may sound like:                     <ul style="list-style-type: none"> <li>"You have two blocks. Can you stack them like this?"</li> <li>"Wow! You put three rectangular prisms together to make an arch."</li> </ul> </li> </ul>
Snack and Shared Mealtimes	<ul style="list-style-type: none"> <li>Encourage children to engage with the trajectory during mealtimes. Include a balance of teacher-led questions and peer-led discussion.</li> <li>When appropriate, allow children to engage with their food to see if they can combine the pieces. This activity may be presented after the child has had plenty of time to eat as not to distract.                     <ul style="list-style-type: none"> <li>"Your crackers are stacked in a tower!"</li> <li>"Hmm, the grapes can't be stacked. Why not?"</li> </ul> </li> </ul>
Outdoor Time	<ul style="list-style-type: none"> <li>Highlight Composing 3D Shapes outside of the classroom during outdoor experiences, highlighting how math exists in the natural world. This can be through structured activities or simply through free play. Always be sure to follow the child's lead.</li> </ul>

# Let's Stack Blocks! [STEMIE]

## *ACTIVITY TYPE: PLAY*

### *Quick Description:*

Through natural play, modeling and repetition, children will learn about 3D shape composition by stacking blocks, lining them up, and building bridges. See materials for additional materials to meet children's unique needs.



## Same Shape Stacker Line Maker Stacker

### **Directions:**

#### Preparing

##### Fine Motor:

Use bigger and/ or fabric blocks that are easier to grasp for children with fine motor delays. Encourage pointing and the use of the pincer grasp (holding an object with the thumb and pointer finger). Model these actions and ask the child to repeat them.

##### Gross Motor:

Some children may be more comfortable at a table during



<b>[LT]<sup>2</sup></b> Trajectory: Composing 3D Shapes Levels: Stacker; Line Maker; Same Shape Stacker	<b>Let's Stack Blocks!</b>	<b>Science:</b> <ul style="list-style-type: none"> <li>Gravity</li> <li>Force and Motion</li> </ul> <b>Technology:</b> <ul style="list-style-type: none"> <li>Sequencing: Follows 1-step directions</li> <li>Repetition: Intentionally does something more than once</li> </ul> <b>Engineering:</b> <ul style="list-style-type: none"> <li>Create: Repeats actions to make things happen</li> </ul>
<b>Unit Objective:</b> Children will explore foundational geometry concepts of 3D shapes and 3D shape composition.  <b>Lesson Objective:</b> A properly set up and maintained block area is necessary for helping children to establish beginning geometry concepts. Through modeling and repetition, children will learn about 3D shape composition by stacking blocks, lining them up, and building bridges.  <b>Age:</b> 2-3 <b>Grouping:</b> Center		
<b>Beginning of the Lesson (Engage):</b>		<b>Vocabulary</b>
<b>Materials:</b> <ul style="list-style-type: none"> <li>Blocks are smooth sided, not interlocking, and there are several types (hollow cardboard or plastic, wooden, cloth, and foam)</li> <li>There are different block shapes – a classic set includes cubes, prisms, spheres, hemispheres, cones, and cylinders</li> <li>There are enough blocks for at least 3 children to play, approximately 30+</li> <li>There is a large, flat space that is protected from classroom traffic</li> <li>There are at least two types of accessories: transportation, people, animals, etc. <ul style="list-style-type: none"> <li>There should be several choices within each of these types. If transportation is an accessory, there are three cars, three trucks, and a perhaps a construction vehicle</li> <li>Safety note: be sure the accessories are not a choking hazard</li> <li>Diversity note: if including people as accessories, this is a great place to show differences in race, gender, gender roles, and ability</li> </ul> </li> <li>Block Maps, if using (see Supports for Differentiation)</li> <li>Empty cardboard boxes can be used as giant blocks. Have children stack and crawl into and through them.</li> </ul>	<b>Preparing the Environment for Access</b>  <b>Fine Motor:</b> Use bigger and/ or fabric blocks that are easier to grasp for children with fine motor delays. Encourage pointing and the use of the pincer grasp (holding an object with the thumb and pointer finger). Model these actions and ask the child to repeat them.  <b>Gross Motor:</b> Some children may be more comfortable at a table during block play. If this is the case, ensure the table is big enough, free of other items, and the appropriate height. Ensure children are appropriately seated, using adaptive positioning if necessary.	Manipulation <ul style="list-style-type: none"> <li>Stack</li> <li>Hold</li> <li>Drop</li> <li>Knock over</li> </ul> Size <ul style="list-style-type: none"> <li>Big</li> <li>Small</li> <li>Tall</li> <li>Short</li> </ul> Spatial Words <ul style="list-style-type: none"> <li>In/ out</li> <li>On top/ on bottom</li> <li>Beside</li> <li>Beneath</li> </ul>

<b>[LT]<sup>2</sup></b> <b>During the Lesson (Explore, Observe, Extend):</b>		
<b>Instruction</b>	<b>Supports for Differentiation:</b>	

<ol style="list-style-type: none"> <li>Exploring Stacking <ul style="list-style-type: none"> <li>Model stacking by slowly and deliberately placing blocks on top of each other one at a time. Make sure the surface is hard, flat, and even to make stacking easier. This can be a hard floor, a table, or a board.</li> <li>Encourage accuracy (lining up a new block with the one before it) in stacking by telling the child to build as tall of a tower as they can.</li> <li>As children advance through the levels, they will realize that shapes stack better when they are the same, and if they are different, it's easier to build a tower with bigger blocks on the bottom and smaller blocks at the top.</li> <li>What you say: <ul style="list-style-type: none"> <li>Explore the difference between curved and flat. Try to stack spheres while narrating: "Can I make a tower out of these balls? Oh, no! They all fall down."</li> <li>"Watch how I can put one cube one top of another. Can you do that?"</li> <li>"Oh, you built a very tall tower! What happens if you push it?"</li> <li>"Your tower fell over! Let's build it again!"</li> </ul> </li> </ul> </li> <li>Exploring Building and Line Making <ul style="list-style-type: none"> <li>This is great time to add accessory toys (cars, people, animals; see Materials section above) if you</li> </ul> </li> </ol>	<b>Environment</b> <ol style="list-style-type: none"> <li>If children are more comfortable doing block play at a table, ensure the table is big enough, free of other items, and the appropriate height.</li> <li>Allow children to sit, stand, lay as needed to access materials.</li> </ol> <b>Materials</b> <ol style="list-style-type: none"> <li>Use bigger and/or fabric blocks for easier grasping.</li> </ol> <ol style="list-style-type: none"> <li>Instruction <ol style="list-style-type: none"> <li>Introduce vocabulary and model language around block play.</li> <li>Give plenty of wait time to let the child respond to questions and prompts.</li> </ol> </li> </ol>
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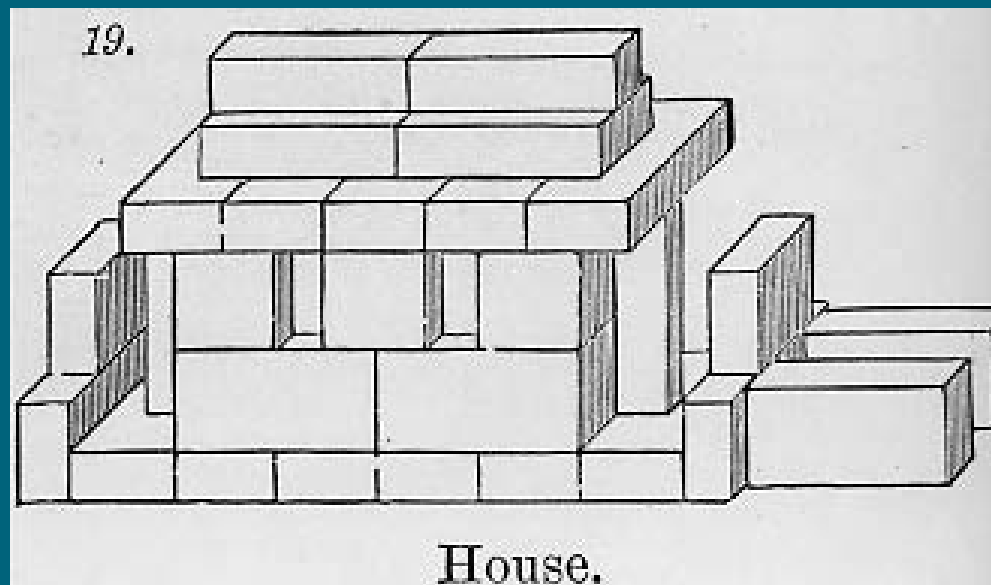
# Block Building

*Why* LTs?



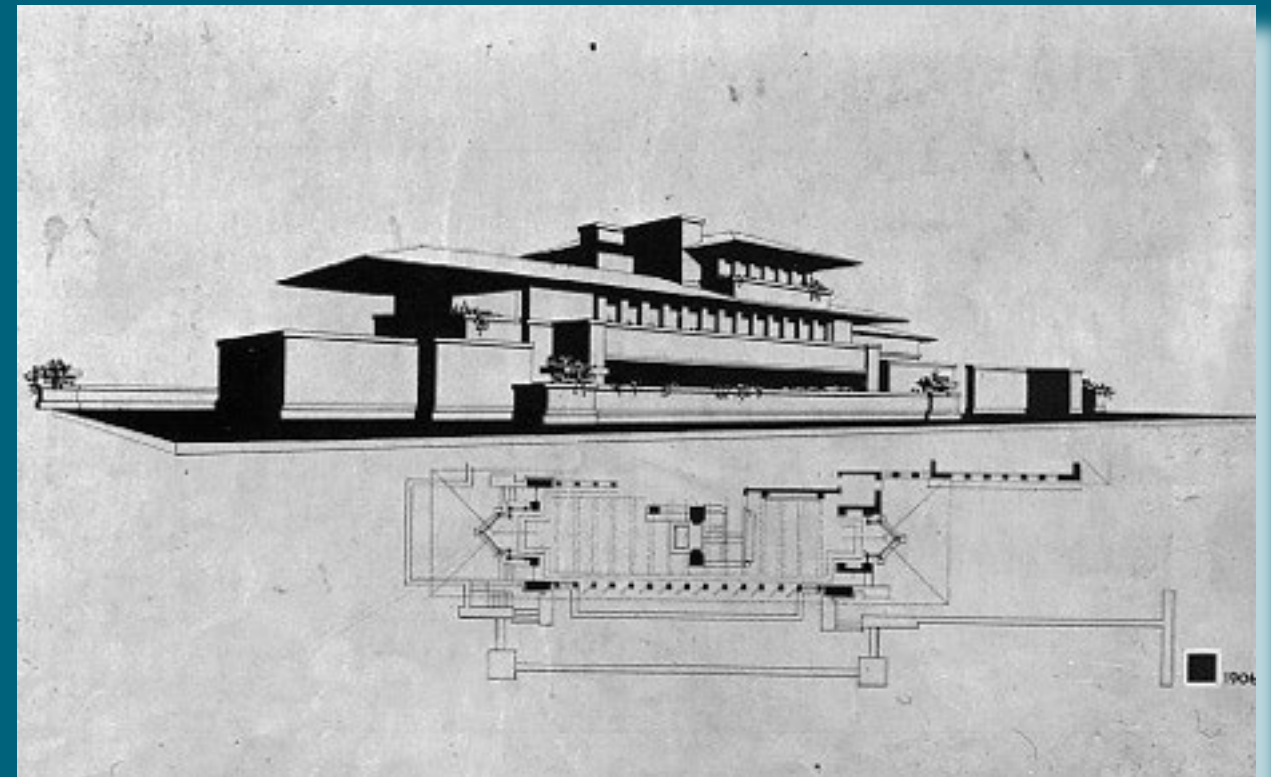
*How* use LTs?

# Frank Lloyd Wright



6th gift, 1877

Wright, Robie House, 1906



So, solving problems  
with intentional teaching!



# Contributions

- Practice-based evidence of success, scalable
- Clear guidelines
  - Asset-based, grounded in knowledge of children's thinking and learning: LTs
  - Children engaged, active, inventive, talking
  - Content that is challenging but achievable...
- Now: [LearningTrajectories.org](https://LearningTrajectories.org)

